Free Vibration and Buckling Analysis of Tapered Beam with Open Transverse Crack

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ABSTRACT

Tapering beams are used in diversities for their economic, aesthetic and other considerations in architecture, aeronautics, robotics and other innovative engineering applications. More recently they have been the subject of numerous studies. Present study deals with the vibration and buckling behavior of linearly tapered beams with single open transverse cracks. The variety of natural frequency and buckling load with distinctive parameters including relative crack depth, position of cracks and the slope of the beam are analyzed using finite element methods (FEM). A Matlab code is developed for the computation of natural frequencies and buckling loads of the cracked beam. Crack in the beam is represented by a rotating spring in line with T. D. Chaudhari, S. K. Maiti (1999). Stiffness matrix of the cracked tapered beam element is obtained from the flexibility matrix of the intact beam and the additional flexibility matrix due to crack. Free vibration frequencies and buckling load of a cracked cantilever beam reduce with an increase in crack depth. And also frequencies reduce more with the crack located nearer to the fixed end than the free end. For an intact tapered beam the frequencies vary more for the depth ratio compared to breadth ratio. The free vibration frequencies of a single cracked beam also vary more for depth ratio compared to breadth ratio. The vibration results can also be utilized as a tool for structural health monitoring, testing of structural integrity, execution and safety.

INTRODUCTION

It is known that beams are the basic structural components and can be classified according to their geometric configuration. They are usually uniform or non-uniform, and slender or thick. Non-prismatic members are increasingly being used in diversities as for their economic, aesthetic, and other considerations. . In this study modeling of a tapered beam of linearly variable depth and constant thickness with crack normal to the axis is performed by FEM in Matlab environment. In the 19th century, Russian engineer D.I. Zhuravskii first deduced a formula to calculate shear stresses in prismatic beams with rectangular cross-sections based on the static equilibrium in a small element of a prismatic beam and the theorem of conjugate shearing stress. The resultant formula is the well-known Zhuravskii shear stress formula found in most mechanics of materials textbooks. However, the Zhuravskii shear stress formula is limited by the fact that it is only applicable to rectangular prismatic beams within the elastic range.

When used in non-prismatic beams, the Zhuravskii shear stress formula will cause substantial errors. Accordingly, Gere and Timoshenko [1] proposed an analytical expression for a tapered cantilever beam with a rectangular cross-section under both moment and shear, from which they found that the distribution of shear stress in a non-prismatic beam is quite different from that in a prismatic beam. Furthermore, Zhou et al.[2]

Derived a general formula to calculate shear stresses in non-prismatic beams with corrugated steel webs subjected to combined axial forces, moment and shear. Paglietti and Carta [3] also proved that the conventional formula for tapered beams is incapable for that the shear stress is not maximum at the centroid of the sections if the beam is of a variable depth.

Chong et al. [4] investigated the shear stress distribution of tapered beams by using a theory based on linear elasticity. A simple and efficient beam model was proposed by Balduzzi et al. [5] to investigate the influence of variable cross-sections on beam behaviours, and they also noted that the shear stress depends not only on the sheer force but also on the axial force and bending moment. Based on finite element analysis, El-Mezaini et al. [6] revealed that the discontinuity of the centroidal axes in non-prismatic beams could cause a strong coupling among the moment, shear and axial forces, which illustrates the distinct difference between non-prismatic and prismatic members.

According to the above studies, there are great differences in the mechanical behaviours of prismatic and non-prismatic members.

LITERATURE REVIEW

Bazoune et. al. (2001) used the finite element method to develop a method for dynamic response of spinning tapered Timoshenko beam. They considered the effects of Coriolis forces, rotary inertia, shear deformation, angular setting, taper ratios and hub radius of the beam while developing the equations of motion. The values obtained by this method are less accurate.

Radha Krishnan (2004) studied the resonance response of a cracked cantilever beam of rectangular cross section. The method is based on fracture mechanics quantities like stress intensity factor, strain energy release rate, and compliance. With the increase in crack length the fundamental frequency decreases, thus stiffness also decreases. They showed that when the amplitude of vibration increases the natural event of resonance gets shifted with increase in length of the crack.

Behzad et. al. (2005) developed the equations of motion with the corresponding boundary conditions for free bending vibration of a beam in the presence of an open edge crack. They used the Hamilton principle for this implementation. The crack has been demonstrated as a continuous disturbance function in displacement field which could be acquired from fracture mechanics.

Kukla and Zamojska (2006) applied green's function method to a free vibration problem of a system of non-uniform beams coupled with non-homogeneous elastic layers. The frequency equation is obtained by using a quadrature rule of a Newton-Cotes type.

Bayat et. al. (2010) published their journal on analytical study of tapered beam vibration frequencies. The considered represents the governing equation of the nonlinear, extensive amplitude free vibrations of tapered beams. They actualized another system called Homotopy Perturbation Method (HPM) over the antiquated Chinese technique called the Max-Min Approach (MMA).

Cheng et. al. (2011) studied the vibration characteristics of the cracked rotating tapered beam are explored by the p-version finite element method. The shape functions enhanced with the shifted Legendre orthogonal polynomials are employed to represent the transverse displacement field within the rotating tapered beam element. The crack element stiffness matrix and the p version finite element model of the basic framework are gotten by utilizing fracture mechanics and the Lagrange equation, respectively.

Trahair (2014) Analyzed the elastic in-plane bending and out of-plane buckling of indeterminate beam structures whose members are having tapered and of mono-symmetric I cross-section with efficient finite element method. Tapered finite element formulations are created by numerical integration rather than the closed forms. The common approximation in which tapered elements are supplanted by uniform elements is indicated to converge gradually, and to prompt off base forecasts for tapered mono-symmetric beams.

Modelling of a Tapered Beam

The beam element is assumed to be associated with two degrees of freedom, one rotation and one translation at each node. The location and positive directions of these displacements in a linearly varying tapered beam element are demonstrated in fig below. Some commonly utilized cross-sectional shapes of beams are demonstrated in Table. The depths of the cross sections at the ends are represented by h1 (at fixed end) and h2 (at free end), similarly the widths at the ends are represented by b1 (at fixed end) and b2 (at free end) respectively. Length of the beam is taken as „l". The axis about which bending is assumed to occur is demonstrated by a line in the center coinciding with the neutral axis.

Finite Element Formulation Materials

Finite element method is the most suitable technique for digitalized computers. It includes a body to be discredited into smaller bodies having equivalent system. Then the whole body is represented by assembling such small bodies. Every subsystem is comprehended separately and the outcomes so acquired are then joined to get solution for the entire body. The finite element method is relevant to the extensive variety of problems, including nonlinear stress- strain relations, non-homogeneous materials and confounded boundary conditions. Such problems are typically handled by one of the three methodologies, namely,

1. Displacement Method or Stiffness Method
2. Equilibrium Method or Force Method
3. Mixed Method

Calculation of Shape Function:

The tapered beam element is assumed to be with two degrees of freedom, one rotation and one translation at each node, as shown in figure 1

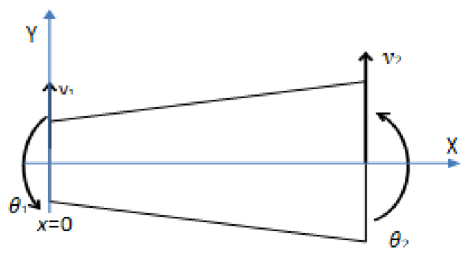

Figure 1 Schematic diagram of a tapered beam element with 2 degrees offreedom per node

RESULT

The dynamic and static behavior of a beam can be studied with its stiffness properties. Structural defects are origin for local flexibilities results deficiency in structural resistance. The presence of cracks in a structure results, changes in its stiffness. We can observe the changes in the local flexibilities Structural deficiencies like cracks give deficiencies in the local flexibilities.

Convergence Study

A graph is plotted between fundamental natural frequencies to the number elements. We can observe from figure 2 that the frequency values are converging at minimum elements of 14

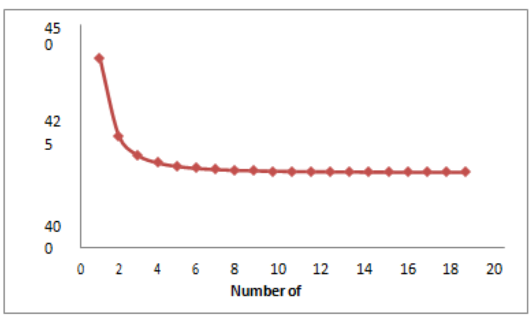

Figure 2 Convergence of fundamental frequency of an intact tapered cantilever beam

Effect of Taper Ratio on Frequency for an intact tapered beam

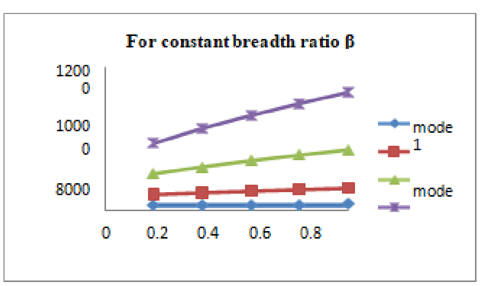

Figure 3 Effect of α on frequency for constant values of β

CONCLUSION

Free vibration and buckling analysis of a tapered beam subjected to a transverse crack has been carried out using Finite element method in Matlab environment. The following observations are concluded from the present study,

- Mathematical formulation for free vibration and buckling analysis of a tapered beam with transverse open edge crack is presented.
- Free vibration frequencies for both intact and single cracked tapered beams increase with increase in depth ratio (α) whereas the breadth ratio (β) has a detrimental effect. This is a very useful concept that can be used in structures or machine members where strength to weight ratio is important to be considered for minimal weight and highest strength, simultaneously increasing the fundamental frequency.
- The natural frequencies for a single cracked taper beam are influenced by crack depth, location of the crack and taper ratio.
- Buckling loads for both intact and single cracked beam s increase rapidly with increase in the depth ratio α than breadth ratio β.
- For a Fixed-Free tapered beam subjected to a single transverse crack, the maximum drop in the buckling load is in the location factor of 0.65 from the fixed end. This is due to the combine effect of strain energy and tapering effect.
- For a Hinged-Hinged tapered beam subjected to a single transverse crack, the maximum drop in the buckling load is in the location factor of 0.78 from the larger end.
- For a Fixed-Fixed tapered beam subjected to a single transverse crack, the maximum drop in the buckling load is in the location factor of 0.7 from the fixed end.
- For a Fixed-Hinged tapered beam subjected to a single transverse crack, the maximum drop in the buckling load is in the location factor of 0.81 from the fixed end.
- With the study of vibration of the tapered beam, possible detection of the crack can estimated.

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